

Exam in Public Finance - Spring 2012  
3-hour closed book exam  
(Answers)

Part 1: Challenges in achieving a social optimum

(1A) The underlying idea of a social welfare function is to map individual preferences/utilities  $(u_1, \dots, u_n)$  into an aggregate utility function  $W(u_1, \dots, u_n)$  that may be used to rank different allocations in society. If the aggregate utility level  $W$  is larger for allocation  $\mathbf{x}$  than for allocation  $\mathbf{y}$  then allocation  $\mathbf{x}$  is a better outcome for society than allocation  $\mathbf{y}$ . A crucial question is whether it is possible to aggregate individual preferences into a social preference that would then reflect "the will of the people". That is, is it logically possible to aggregate preferences of individuals to make some decision that will affect the welfare of everyone in a way that will result in rational social choices that accurately reflect the true preferences of the individuals in society? Arrow's impossibility theorem shows under mild conditions that this is impossible (students may list/discuss the underlying conditions and maybe describe the Condorcet paradox). This is discussed more thoroughly in Sections 10.3, 12.7 and 12.8 of the textbook Hendricks and Myles (2006).

(1B) An externality is present whenever some economic agent's welfare is directly affected by the action of another agent. If it is a global externality then the action of an agent directly affects the welfare of all other agents in the world. Without some sort of global regulation policy, it is difficult to attain the first best allocation. Optimal country-specific regulation policy where the policy maker optimize the welfare of its citizens will result in insufficient regulation because the policy maker only takes into account the adverse externality effects of an agent on the country citizens, and not on everybody else in the world. This may be illustrated graphically. Students may also discuss the second-best optimal country-specific regulation policy when global regulation exists (say, in the form of a tax on externality generating goods or a tradable permit system) but is insufficient. All this is described in greater detail in the teaching note Kreiner (2012).

(1C) The delegation of the provision of local public good to local policy makers may enhance economic efficiency because local policy makers may be better informed about local preferences and cost structures and thus are better able to set the optimal level of local public goods and taxation. Moreover, once the differences in local tax-consumption packages has been establish aggregated welfare may increase further as individuals settle in the municipalities that best match their preferences.

A potential problem with delegation to local policy makers within a fiscal federation is tax externalities. A higher tax rate in a municipality may induce individuals to reduce income, which reduces the tax revenue of the central government (vertical externality). A higher tax rate in one municipality may induce some individuals to move to another municipality and thereby increase their tax revenue (horizontal externality). The municipalities may therefore end up in tax competition. All this is described in greater detail in the teaching note Sogaard (2012).

## Part 2: Responses to taxation and taxes on high incomes

(2A) The individual maximizes utility subject to the budget constraint. This may be solved by setting up a Lagrangian function and differentiate with respect to  $z$ . An alternative is to insert consumption, isolated from the budget constraint (which holds with equality because of monotone preferences), into the utility function:

$$u = \mu(z - T(z)) - \frac{\varepsilon}{1 + \varepsilon} z^{\frac{1+\varepsilon}{\varepsilon}}.$$

The first order condition with respect to  $z$  is

$$\mu(1 - T'(z)) - z^{\frac{1}{\varepsilon}} = 0.$$

Isolation of taxable income gives:

$$z^* = [\mu(1 - m)]^\varepsilon, \quad (1)$$

where  $m \equiv T'(z^*)$  is the marginal tax rate.

The elasticity of taxable income with respect to the after-tax-rate  $(1 - m)$  is defined as

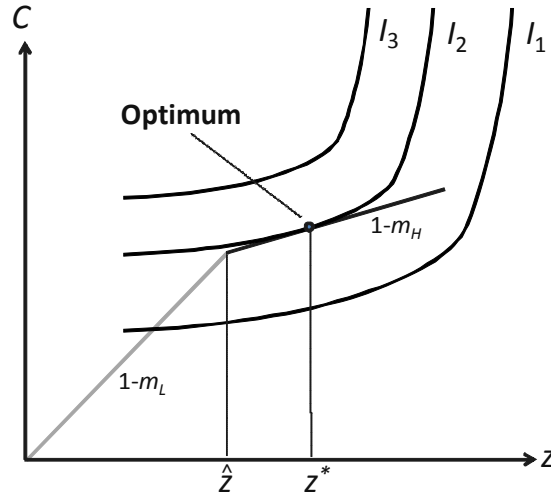
$$\text{ETI} \equiv \frac{dz^*}{d(1 - m)} \frac{(1 - m)}{z^*}$$

From the result above, we obtain  $dz^*/d(1 - m) = \mu\varepsilon [\mu(1 - m)]^{\varepsilon-1}$

$$\text{ETI} = \mu\varepsilon [\mu(1 - m)]^{\varepsilon-1} \frac{(1 - m)}{[\mu(1 - m)]^\varepsilon} = \varepsilon.$$

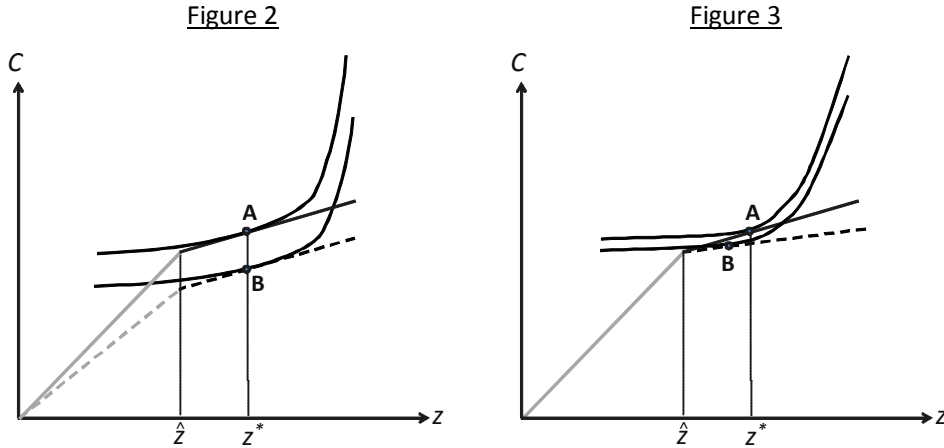
(2B) The budget set, indifference curves and optimal choice are illustrated in Figure 1 (see also Section 15.3 in Hendricks and Myles (2006)). The consumption level at  $\hat{z}$  is equal to  $(1 - m_L)\hat{z}$ . This corresponds to the first part of the budget line with a slope of  $1 - m_L$  until  $\hat{z}$ , after which the budget line is more flat with a slope of  $1 - m_H$ . We know that  $z^* > \hat{z}$  implying that the optimal choice lies at the black, flat part of the budget line.

Figure 1



(2C) An increase in the tax rate  $m_L$  is illustrated in Figure 2 where the dashed curve is the new budget line. The optimum moves from point **A** to point **B**. The higher tax rate reduces the slope at the budget line at  $z < \hat{z}$ . For  $z > \hat{z}$  the slope is still  $1 - m_H$  but the level of consumption is lower because of the higher tax payment. The optimum choice of income  $z^*$  is unchanged, which is seen from (1). The reason is that the individual has quasi-linear preferences implying no income effects on labor supply.

An increase in the tax rate  $m_H$  is illustrated in Figure 3 where the dashed curve is the new budget line. This affects the marginal incentive to earn income at  $z^*$  implying that the individual reduces income due to a substitution effect. Hence, the new optimum at point **B** lies to the left of the initial optimum at point **A**.



(2D) After inserting the budget line and the tax function, household utility becomes

$$u = \mu(z^* - m_L \hat{z} - m_H(z^* - \hat{z})) - \frac{\varepsilon}{1 + \varepsilon} (z^*)^{\frac{1+\varepsilon}{\varepsilon}},$$

where  $z^* = [\mu(1 - m_H)]^\varepsilon$ . This gives

$$\frac{du}{dm_L} = -\mu \hat{z}$$

and

$$\begin{aligned} \frac{du}{dm_H} &= -\mu(z^* - \hat{z}) + \left[ \mu(1 - m_H) - (z^*)^{\frac{1}{\varepsilon}} \right] \frac{dz^*}{dm_H} \\ &= -\mu(z^* - \hat{z}) \end{aligned}$$

where the last equality follows from the first order condition (follows also directly from the envelope theorem).

The tax revenue equals

$$T = m_L \hat{z} + m_H(z^* - \hat{z})$$

The derivative with respect to  $m_L$  and  $m_H$  equal:

$$\frac{dT}{dm_L} = \hat{z} \tag{2}$$

$$\begin{aligned} \frac{dT}{dm_H} &= (z^* - \hat{z}) + m_H \frac{dz^*}{dm_H} \\ &= (z^* - \hat{z}) - m_H \mu \varepsilon [\mu(1 - m_H)]^{\varepsilon-1} \\ &= (z^* - \hat{z}) - \varepsilon \frac{m_H}{1 - m_H} z^*, \end{aligned} \tag{3}$$

where we have used eq. (1). The effect on utility is equal to the mechanical tax change, i.e. the extra tax burden of the individual at the initial optimum (multiplied by the marginal utility of consumption). Behavioral responses of the individual do not affect utility (the individual has chosen a utility maximizing level of  $z$  initially and marginal changes in behavior around this optimum has no first order effect on utility). In the case of an increase in  $m_L$ , individual behavior is unchanged, and the increase in tax revenue is simply equal to the mechanical tax change ( $dm_L \cdot \hat{z}$ ). In the case of an increase in  $m_H$ , the agent reduces income  $z^*$ , and this behavioral response reduces tax revenue, as reflected in the second term of expression (3). The strength of this effect on government revenue depends on the elasticity of taxable income  $\varepsilon$  and the initial level of the marginal tax rate  $m_H$ .

**(2E)** The tax rate  $\tilde{m}_H$  is the level that maximizes tax revenue. A tax rate above this level will reduce tax revenue because the negative behavioral effect on government revenue becomes large than the positive mechanical effect on government revenue. As is apparent from the formula and the table,  $\tilde{m}_H$  is decreasing in  $\varepsilon$  and  $\alpha$ . A larger elasticity  $\varepsilon$  gives a larger negative behavioral response, because the individual is more responsive to the economic incentive, and therefore a lower  $\tilde{m}_L$ . A higher  $\alpha$  implies that the positive mechanical effect of a tax increase becomes small relative to the behavioral effect. The reason is that the tax rate  $m_H$  only applies to income above  $\hat{z}$ . If, for example,  $z^*$  is just slightly above  $\hat{z}$  then the positive mechanical effect is close to zero.

[Students may also recall from the lecture slides that  $\alpha$  is around 3.4 for Denmark and around 1.8 for the US. From the table this implies that the revenue-maximizing tax rate is lower in Denmark than in the US. For  $\varepsilon = 0.2$  (the estimate for high-wage earners by the Danish Economic Council, 2011), the revenue-maximizing tax rate is then around 60 percent in DK but maybe around 74% in the US because of differences in  $\alpha$ . The revenue-maximizing tax rate is rather sensitive to the elasticity of taxable income  $\varepsilon$  and there is a lot of uncertainty with respect to the size of the elasticity. The table shows that the revenue-maximizing tax rate varies from 50 percent to 75 percent for realistic elasticities for  $\alpha = 3.4$ .]

[Students may also note that the formula for  $\tilde{m}_H$  may be derived by setting the expression in (3) equal to zero.]

### Part 3: Incidence and empirical measurement

**(3A)** The firm is remitting the tax to the tax authorities and has therefore the formal tax incidence. The formal/legal tax incidence describes who has the legal obligation to pay the tax. The economic incidence describes the economic burden of the tax on the different agents, which may differ from the formal tax incidence. (In fact, economic tax incidence does not depend at all on the legal tax incidence.)

**(3B)** The economic incidence is determined by the relative size of the elasticities. The part with the lowest elasticity will bear most of the economic burden. Consider, for example, the case where labor supply is completely inelastic,  $\varepsilon_w = 0$ . In this case, workers supply a fixed number of work hours, completely independent of the after tax wage rate. A higher tax on firms will reduce labor demand but, because the labor supply curve is vertical, this will go directly into a lower wage of workers. In the other extreme case, labor supply is perfectly elastic,  $\varepsilon_w \rightarrow \infty$ , implying that workers ask for a fixed wage rate independent of demand. In this case, firms will reduce demand until the point where they are willing to pay workers the original wage plus the tax, and firms will therefore bear the full burden of the tax. These points may be illustrated graphically.

If the elasticity of labor demand is completely inelastic,  $\varepsilon_F = 0$ , or labor supply is perfectly elastic,  $\varepsilon_w \rightarrow \infty$ , then firms bear the full burden of the tax and the economic incidence will therefore coincide with the formal incidence.

**(3C)** The empirical analysis in Gruber (1994) shows that the extra health insurance costs of the firms due to mandated maternity benefits are shifted to the employees through lower wages. According to the estimate in Table 3, hourly wage rates decrease on average by 5.4 percent of those affected. The overall conclusion is that the degree of shifting is significantly different from 0% but not significantly different from 100%.

The empirical analysis exploits that some states passed laws prohibiting treating pregnancy different from other illness and thereby imposing higher health insurance costs of firms. This creates exogenous variation in labor costs of fertile women compared to other groups in the states passing the laws (within state variation) and also exogenous variation in labor costs of fertile women living in the states passing the laws compared to fertile women living in other states (across state variation). Gruber constructs a diff-in-diff-in-diff estimator that exploits the exogenous variation in both dimensions at the same time. He first compute the after-before reform difference in wages of fertile women in the treatment states relative to the control states and he then subtracts the same diff-in-diff computed for other individuals not subject to the reform (and arrive thereby at the result of 5.4 percent reported in the bottom of the table). This method controls for state-specific time trends (within-state difference) and group-specific time trends (between-state difference). The identifying assumption is therefore identical trend-differences between fertile women and others in treatment states and control states without the reform. This is a weaker assumption than the common trend assumption underlying the standard diff-in-diff identification strategy and is possible because of the two dimensions of variation in the data.